Eigen-sets of curvature measure as a technique for defining separability among adjoining regions of n-dimensional spaces

Exploratory Notes

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We address from a novel perspective the problem of distinguishing regions within an n-dimensional space when those regions may be adjoining, overlapping, and (from one instance of observation to others) undergoing nonlinear and unpredictable transformations affecting their individual and collective geometries. We believe that there may be methods for more efficiently and accurately rendering these regions into identifiable entities, each of which will consistently maintain some characteristics, qualitatively analogous to an eigenfunction but drawing upon a set of flow or gradient measures related to changes in curvature taken cumulatively from multiple segments of the region surfaces, that will maintain stability and distinguishability from those sets of neighboring regions with which they could otherwise be confused.

If our investigations can be extended and shown to have merit, then this may open up a new pathway within mathematics and computational analysis that can be of value in many areas of current research, such as within image processing, surface and subsurface sensing, financial and psychohistorical trend forecasting, meteorology, cosmology.

Consider four unique bundles of objects. The reason for these four types is because of their characteristics and utility for visualization as well as for the fact that we will consider later some transformations between their types. Each bundle can be thought of as a cluster of several objects which will represent our regions in the n-space of interest. Initially, we will limit our discussion to an ordinary 3-space, and specifically a Euclidean 3-space. However, what follows may be generalizable to other, non-Euclidean spaces.

Bundle type I is a cluster of billiard balls. Each ball is the same size, a regular 2-sphere.
Bundle type II is a cluster of common fruits – oranges, apples, pears, limes.
Bundle type III is a cluster of clumps of artists' modeling clay or the "Play-Do" used by children. We shall assume that the clumps are relatively similar in size, ranging from 5 to 15cm in diameter for a 2-sphere that would contain a given clump if it were molded into a mainly spherical shape.
Bundle type IV is a cluster of water balloons, sufficiently loose-filled so as to be very easily changed in shape from light pressure applied by hand or by pressure from adjoining balloons.

Each object in a bundle defines a region and the bundle as a whole represents the space in which all of the regions co-exist. Note that the space is not completely filled by the regions as there will be gaps among them, but for purposes of this exploration we are ignoring the space beyond the boundaries of the bundle; it is not important for the problem at hand.

Figures 1.1 through 1.4 illustrate the basic "laboratory toolset" for our exploration.

[Here - have to draw them or use photographs – Later]

First, we want to consider the range of dynamic behaviors that can take place within each of these four bundles. Next, we will examine phenomenologically the methods by which a human observer can make distinctions – and also mistakes – in the visual separation of each object

within a given cluster, as it remains static and also as it undergoes changes that affect the topology of the individual objects and the cluster as a whole.
Third, we will attempt to establish a formal model for describing the ways in which the curvatures of the regions can be described as sets.

With this formal model we will attempt to make some assertions and to derive some rules and heuristics that can be used in the course of pattern discrimination and recognition. There is one basic assertion that we claim to be interesting and worthy of investigation, in part for purely theoretical interest but also for value in applied sciences.

Assertion 1:
Given a collection of n space-filling regions $\{r_1, \ldots, r_n\}$ that are amorphous and also contiguous to one another, thus comprising a bundle within which each region r_i maintains contact with at least one other region r_j, during any transformation of position or deformation of its local (regional) topology, there is preserved a relationship between the measurable changes in curvature among the collection of regions that can be associated with individual regions to unique identify those regions during the course of all transformation sequences so long as the volumetric integrity of each region is not destroyed. This may be termed "conservation of curvature" and it can be used to separate and track individual regions as they interact and mutually deform one another in the space that they occupy and fill.